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A Model for the Use of Facilities for the Treatment of Addiction

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Abstract

In this study, considering the importance of treatment of addiction as an illness, we have analyzed a model of harvesting of facilities for the rehabilitation and treatment as a new application of game theory. According to the results of this research, it is possible to avoid a waste of money, energy, and facilities with a better management of allocating facilities and we can treat more addicts.

Keywords: Game theory, Dynamic systems, Harvest function, Statistical mechanics.

1. Introduction and preliminaries

Static games with complete information have many applications in economics, politics, sociology, etc.. [2, 6, 7, 10, 11].
These applications include the study of the exploitation of renewable natural resources and different harvesting strategies, particularly in fshing Management [1, 3, 4, 9, 13].
Of course, in the field of fishing, many studies have been done in Dynamic Systems [5, 8, 12, 14].
Since addiction is known as a disease, treatment of this disease requires facilities such as rehabilitation camps, hospitals, and even some special drugs, entertainments, Part-time and alternative jobs as well as experts. By getting the idea of a combination of Dynamic Systems and Game Theory to model the problem of fishing in harvesting of renewable natural resources, we present the following model to investigate the use of addicts from rehabilitation facilities. We will use $X(t)$ to show the facilities and services that we will give to the community of addicts and r is the net reproduction rat of each unit of these facilities. Due to the limited funding available for supplying and charging these facilities over the period under consideration, K is the carrying capacity. It should be noted that if, along

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with the rate r, the amount of facilities is greater than K since there is no budget for reproduction, these facilities would get out of the cycle so the amount of facilities would get back to level K and if the amount of facilities were lower than K the facilities would again reach the level of K in the end, because in that way the essential amount of budget would be available. It is essential to say that in the beginning of the course, the growth of the facilities would speed up and it is positive, but in the passing of time these facilities would increase and on the other hand the expense of providing these facilities would also increase, so the level of growth changes would decrease but these changes would still be positive and they would get closer to the maximum amount , but in passing of time the amount of facilities would increase, because of this the expense would increase so the growth of facilities would be positive before reaching level K but it would be descending and by reaching to level K the changes of the facilities would be zero. Also if the facilities do not be used , the amount of these facilities would catch level K and it will be stopped there. According to explanation above we can show the relation between the growth of facilities and the carrying capacity by using logistic growth function in other words

$$L = \frac{dX}{dt} = rX(1 - \frac{X}{K}).$$

But the addicts must have the motivation to use these facilities and they must make effort to use them. In order to use these facilities, we consider the density-dependent harvesting of $H(t) = qEX(t)$ in which E is the effort to use these facilities and $q \geq 0$ is the probability of getting these facilities.

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2. Second section

2.1. Model formulation and basic properties

According to the above description, the harvest will change the equilibrium and the dynamics of this model is

$$\frac{dX}{dt} = rX(1 - \frac{X}{K}) - qEX. \tag{2.1}$$

It can be written as follows

$$\frac{dX}{dt} = r_0 x(1 - \frac{X}{K_0})$$

where $K_0 = K(1 - \frac{qE}{r})$ and $r_0 = r - qE$. This system has a trivial equilibrium, $X = 0$, and an non-trivial equilibrium, $X = K_0 = K(1 - \frac{qE}{r}) > 0$ provided that $r > qE$.
Considering $f(X) = rX(1 - \frac{X}{K}) - qEX$, we have $\frac{dX}{dt} = f(X)$. Since $\frac{df(X)}{dX} = (r - qE) - 2\frac{rX}{K}$, then $\frac{df(0)}{dX} = r - qE > 0$ so the trivial equilibrium point is unstable, and at the non-trivial equilibrium $\frac{df(K_0)}{dX} = -r + qE < 0$ which shows the stability of this point.
By solving the differential equation (2.1) by the method of *separation of variables* and taking $B = \frac{X(0)}{|K_0 - X(0)|}$, it follows that if $K_0 - X > 0$ then $N(t) = \frac{BK_0 e^{r_0 t}}{1 + Be^{r_0 t}}$ is the solution of this differential equation and since $\lim_{t\to+\infty} X(t) = K_0$, it implies that the non-trivial equilibrium is asymptotically stable and if $K_0 - X < 0$ then $N(t) = \frac{-BK_0 e^{r_0 t}}{1 - Be^{r_0 t}}$ is the solution of the differential equation and $\lim_{t\to+\infty} X(t) = K_0$ implies that the non-trivial equilibrium is asymptotically stable. When the system reaches at the equilibrium point, we have

$$X = K(1 - \frac{qE}{r}). \tag{2.2}$$

This is a relation between the effort and the facilities in equilibrium and by substituting it in the harvest function, we have

$$H = KqE(1 - \frac{qE}{r}). \tag{2.3}$$

We consider two categories of addicts, U_1 and U_2, as two players that they do a static game with complete information in devoting the amount of effort to harvest the facilities. If E_1 and E_2, respectively, represent the average effort that drug addicts in each group do for the use of facilities then the total effort to harvest from this facilities is $E_T = E_1 + E_2$ and the total harvest of this effort is $H_T = KqE_T(1 - \frac{qE_T}{r})$.
In this model, we assume that the share of each group is equal to its share of total effort in other words

$$H_1 = \frac{E_1}{E_T}H_T = \frac{E_1}{E_T}KqE_T(1 - \frac{qE_T}{r}) = KqE_1(1 - \frac{qE_2}{r}) - \frac{Kq^2E_1^2}{r},$$

$$H_2 = \frac{E_2}{E_T}H_T = \frac{E_2}{E_T}KqE_T(1 - \frac{qE_T}{r}) = KqE_2(1 - \frac{qE_1}{r}) - \frac{Kq^2E_2^2}{r}$$

in which $H_T = H_1 + H_2$.
If each unit of facilities and equipments that uses for the rehabilitation and treatment has a value of P and one unit of effort has a cost C, then the outcome of the players is

$$U_1(E_1, E_2) = PH_1 - CE_1 = KPqE_1(1 - \frac{qE_2}{r}) - \frac{KPq^2E_1^2}{r} - CE_1,$$

$$U_2(E_1, E_2) = PH_2 - CE_2 = KPqE_2(1 - \frac{qE_1}{r}) - \frac{KPq^2E_2^2}{r} - CE_2.$$

By calculating, the best response functions of players are

$$E_1 = B_1(E_2) = \begin{cases} \frac{r}{2q} - \frac{Cr}{2KPq^2} - \frac{E_2}{2} & \text{for} \quad E_2 < \frac{r}{q}(1 - \frac{C}{KPq}) \\ 0 & \text{for} \quad E_2 \geq \frac{r}{q}(1 - \frac{C}{KPq}) \end{cases},$$

$$E_2 = B_2(E_1) = \begin{cases} \frac{r}{2q} - \frac{Cr}{2KPq^2} - \frac{E_1}{2} & \text{for} \quad E_1 < \frac{r}{q}(1 - \frac{C}{KPq}) \\ 0 & \text{for} \quad E_1 \geq \frac{r}{q}(1 - \frac{C}{KPq}) \end{cases}.$$

According to the functions E_1 and E_2, the Nash equilibrium for this model is

$$(E_1^*, E_2^*) = (\frac{r}{3q}(1 - \frac{C}{KPq}), \frac{r}{3q}(1 - \frac{C}{KPq})). \tag{2.4}$$

The total effort in the Nash equilibrium is

$$E_T = E_1^* + E_2^* = \frac{2r}{3q}(1 - \frac{C}{KPq})$$

and the total harvest is

$$H_T = H_1^* + H_2^*$$

where

$$H_1^* = KqE_1^*(1 - \frac{qE_2^*}{r}) - \frac{Kq^2E_1^{*2}}{r} = \frac{Kr}{9}(1 - \frac{C}{KPq})(1 + \frac{2C}{KPq})$$

and similarly

$$H_2^* = \frac{Kr}{9}(1 - \frac{C}{KPq})(1 + \frac{2C}{KPq})$$

then

$$H_T = \frac{2Kr}{9}(1 - \frac{C}{KPq})(1 + \frac{2C}{KPq}).$$

On the other hand, by substituting in (2.3) we have

$$KqE_T(1 - \frac{qE_T}{r}) = \frac{2Kr}{9}(1 - \frac{C}{KPq})(1 + \frac{2C}{KPq}). \tag{2.5}$$

But this equation has $\Delta = \frac{q^2}{9}L(C)$ in which $L(C) = \frac{16C^2}{(KPq)^2} - \frac{8C}{KPq} + 1$. On the other hand $L(C)$ has only a root in $\frac{KPq}{4}$ and this point is a local minimum. Therefore $\Delta \geq 0$ and (2.5) has twe real roots for each $C \neq \frac{KPq}{4}$.
By a simple calculation, these two roots are

$$E_{T_1} = \frac{r}{3q}(1 + \frac{2C}{KPq}), E_{T_2} = \frac{2r}{3q}(1 - \frac{C}{KPq}).$$

That E_{T_2} is the sum of the amount of effort of two players in Nash equilibrium. Also $\Delta E = E_{T_2} - E_{T_1} = \frac{r}{3q}(1 - \frac{4C}{KPq})$.
Corollary. If $C < \frac{KPq}{4}$ then $\Delta > 0$ and this will be a waste of money and facilities, because with a less effort, $\frac{E_1}{2}$, we can achieve the same result as the effort resulting from the Nash equilibrium. So in this case, a static game with complete information will lead to waste of money and facilities. But if $C > \frac{KPq}{4}$ then doing a static game with complete information is suggested to exploit facilities of the rehabilitation because Nash equilibrium provides an optimal option.

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