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Risk Based Portfolio Modeling: Kahneman-Tversky Approach

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Abstract

The risk based portfolio selection problem with investor's behavioral risk aversion bias under uncertainty is monitored. The main results of this research are developed heuristic approaches for the prospect theory model proposed by Kahneman and Tversky in 1979 as well as an empirical numerical analysis of this model with real data. The main purpose is to impose behavioural features of prospect theory to minimize the risk with the certain level of income to the portfolio selection problem. In this research the real data of TSE is employed for computational results with regards of prospect theory model with several stocks as risky assets. In order to investigate empirically the performance of the behaviourally based model, different portfolios are selected with different operating sectors. The aggressive behaviour in terms of returns of the prospect theory model with the specific risk have the same output as returns. In the other hand, the certain level of returns as constrain shows minimized risk with implimentation of behavioral approach in compare to conventional approach which is Markowitz model. The performance of the two behavioral and traditional models are compared and lower risks are obtained by behavioral approach which shows the optimal portfolio selection and might be used by investors as their investment strategy.

Keywords: Portfolio optimization; Behavioural finance; Prospect theory; Risk modelling.

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1. Introduction and preliminaries Literature

The portfolio optimisation problem is to determine an amount of capital to invest in each type of asset in order to receive the highest possible return (or lower risk) subject to an appropriate level of risk (or return) within the portfolio. Modern Portfolio Theory (MPT) began with a paper (Markowitz 1952) and a book (Markowitz 1959) written by the Nobel laureate Harry Markowitz. Many researchers consider the emergence of this theory as the birth of modern nancial mathematics (Rubinstein 2002). The cornerstones of Markowitz's theory are the concepts of return, risk and diversi cation. It is widely accepted (Rubinstein 2002) that an investment portfolio is a collection of income-producing assets that have been acquired to meet a nancial goal. However, an investment portfolio as a concept did not exist before the late 1950s. Remarkably, there is a long history behind the Expected Utility Theory (EUT) that started in 1738 when Daniel Bernoulli investigated the St. Petersburg paradox. The recent financial crisis has shown the shortcomings of the individual market instruments and the low level of validity in investment decisions. This can be explained by the dismissive investors' attitude in assessing the real risks, they usually just follow their own intuition. In the investment practice, the situation of unaccounted risks is fairly common, hence, the investors need to have a reliable mathematical tool for justi cation of investment decisions. In this paper we consider BPT as a tool which takes into account behavioural errors. BPT was developed by Shefrin and Statman in 2000 (Shefrin and Statman 2000). The main idea of the theory is the maximisation of the value of the investor's portfolio in which several goals are met and these goals are considered with di erent levels of risk aversion. BPT is based on two main theories: Security-Potential/Aspiration Theory (SP/A) and Prospect Theory (PT). SP/A theory, established by Lola Lopez in 1987 (see (Lopes 1987)), is a general choice (not only nancial) risk framework and not specied for the portfolio selection problem. It uses two independent criteria of overall utility and aspiration level. In our research we focus on the PT (Kahneman and Tversky 1979) devoted to human behaviour in financial decision making under risk. The main idea from the expected utility theory and adds in the vital psychological components, which take into account human behaviour in the decision making process. It also xes di erent types of inaccuracies that took place in previously developed behaviour based theories, e.g the independence axiom and inconsistency with a uniform attitude towards risk, see (Shefrin and Statman 2000). Loss aversion is a corner stone for prospect theory, especially for portfolio performance evalua-tion (Zakamouline and Koekebakker 2009) and market price of risk (Levy 2010). Since prospect theory was proposed many researchers studied the loss aversion in asset pricing (Barberis and Huang 2001), (Lia and Yang 2013), (Easley and Yang 2015), price volatility (Yang and Wu 2011) and insurance (Wang and Huang 2012) very successfully.

Although the original formulation of prospect theory was only defined for lotteries with two non-zero outcomes, it can be generalised to n outcomes. Generalisations have been used by various authors (Schneider and Lopes 1986), (Camerer and Ho 1994), (Fennema and Wakker 1997), (Vlcek 2006). The original PT choice process consists of two phases. To understand the features of prospect theory let us analyse two approaches to the portfolio selection problem which are traditional (modern portfolio theory) and behavioural (behavioural portfolio theory). We focus more on the assumptions underlying these theories which govern the investor’s choice. Modern portfolio theory uses several basic assumptions namely rational investor", normal distribution of asset returns and neglection of transaction costs (Markowitz 1959). At the same time it was shown that in real life market conditions, these assumptions are not valid (Das et al. 2010), (Mandelbrot 1963) (Fama 1968), (Peng et al. 2008), (Patel and Subrahmanyam 1982), (Fisher and Lorie 1970), (Evans and Archer 1968), (Jacob 1974), (Szego 1980), (Sengupta and Sfeir 1985). The mean variance model which exists in the MPT framework is both general and static for a signi can’t range of practical situations and at the same time it is simple enough for theoretical analysis and numerical solution. This provides widely use of the mean variance model in practice all over the world. However, the portfolio selection problem becomes even more complicated in modern economic conditions which demand more exible and multi-factor models and tools to satisfy the investor’s preferences, while MPT’s assumptions lead to some serious limitations.

There are several reasons why it is not easy to compare MPT and prospect theory approaches. The goal of this paper is to identify potential optimal solution of behaviourally based prospect theory model depending on diferent assets from different sectors situation in comparison with traditionally accepted portfolio optimisation model to minimize the risk. The main contribution of our work is to show the minimization of the risk in portfolio selection with consideration of constant return which the behavioral approach shows the minimized solution in compare to traditional Markowitz approach.

2. Formulation of Prospect Theory Model

The question about the di erence and ratio between the portfolio allocation according to mean variance optimisation and prospect theory utility function optimisation is very challenging in the literature. Many scientists attempt to conceptualise the bene ts and drawbacks of each approach depending on specific market situations, data and assumptions. In this section the formulation of the prospect theory are presented. In the rest of the paper, we will use the following notation:

N - number of assets,
S - number of scenarios (time periods),
K - cardinality limit (desirable number of assets in the portfolio),
p_s - probability of scenario s, $\sum_s P_s = 1$,
r_i - mean return of asset i,
r_{is} - return of asset i in scenario s, i = 1, … ,N, s = 1, …,S,
r_0 - reference point,
$w_i \geq 0$ - weight of asset i in the portfolio,
x = (w1, … ,wN)- a portfolio and $\sum_{i=1}^{N} w_i = 1$
$X = \{x = (w_1, \dots, w_N) \in R_+^N\}$ −set of all, portfolios
$r_{s(x)}$ - return of portfolio x in scenario s,

d - desirable level of return.

It should be noted that one can transfer these models with a cardinality constraint into the basic models if we put *K=N*. For the sake of simplicity we can use formulation for both, basic and cardinality constrained models.

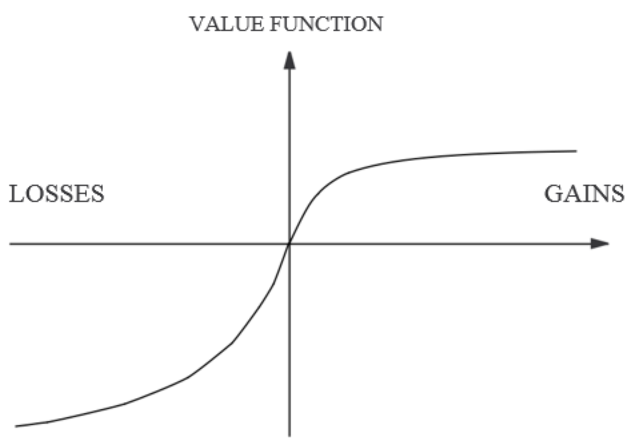

Figure: Prospect theory value function v(r) with α = β = 0.88 and λ = 2.25

where (r_s,p_s), $s = 1,2,..., 0,...,s -1, s,$ means that the gambler wins rs with probability p_s, of course, the sum of all probabilities is equal to *1*, i.e. $\sum_{s=1}^{S} P_s = 1$; r_0 denotes some numerical boundary called the reference point (constant) which depends on the investor's preference. Let r_s define the outcomes of such that:

- if $s = 0$, i.e. $r_s = r_0$, then the investor's gain is 0,
- if $s > 0$, then $r_s > r_0$, hence the investor won from this investment,
- if $s < 0$, then $r_s < r_0$, hence the investor lost.

According to the prospect theory one needs to make additional mental adjustments in the original probability and outcome value functions *p* and *r*, which is equivalent to replacing a standard utility function by the prospect theory utility function. In order to do so we transform the original p and r into the prospect theory probability weight function *π(p)* and value function *v(r)*.
The prospect theory probability weighting function *π(p)* measures, according to (Kahneman and Tversky 1979), "the impact of events on the desirability of prospects, and not merely the perceived likelihood of these events", i.e. expresses the weights of the decisions to the probabilities. Let us mention that *π(p)* is an increasing function, $\pi(0) = 0, \pi(1) = 1$, and for very small values of probability *p* we have $\pi(p) \geq p$. The probability weighting function based on the observation that most people tend to overweigh small probabilities and underweigh large probabilities.
The prospect theory value function *v(r)* describes the (behavioural) value of the gain/loss outcome.

Kahneman and Tversky experimentally obtained the value function which was dependent on the initial value deviation. This function is usually asymmetric with respect to a given reference point r_0 (which reflects different investor's attitude to gains and losses), it is concave upward for gains and convex downward for losses. Moreover, generally the value function *v(r)* grows steeper for losses than for gains, i.e. for $s > 0$ we have $v(rs) \leq -v(r\text{-}s)$.

The explicit formula for the prospect theory value function *v(r)*, given in (Tversky and Kahneman 1992), is:

$$v(r) = \begin{cases} (r - r_0)^{\alpha}, if\ r \geq r_0, \\ -\lambda(r_0 - r)^{\beta}, \quad if\ r < r_0 \end{cases}$$

where $\alpha = \beta = 0.88$ are risk aversion coefficients with respect to gains and losses accordingly, $\lambda = 2.25$ is the loss aversion coefficient which underlines differences in the investor's perception of gains and losses. We note that the value function above is nonlinear with respect to return r and, hence, the portfolio variable *x*. Figure contains the graphs for the value function *v(r)*. The prospect theory utility function can be written in terms of π and v as:

$$PT_U = \sum_{s=1}^{S} \pi(P_s)\, v(r_s) = \sum_{s=1}^{S} P_s\, v(\sum_{i=1}^{N} r_{si} w_i).$$

Clearly, the formula consists of two parts. The part in the gain domain (i.e. when $r \geq r_0$) is concave and the part in the loss domain (i.e. when $r \leq r_0$) is convex, capturing the risk-averse tendency for gains and risk-seeking tendency for losses as seen by many decision makers (Rieger and Wang 2008). Let as mention, that for the sake of simplicity in our study we use $\pi(p) = p$. Clearly, the prospect theory utility function is a nonlinear function.

The prospect theory model aims to find the best (optimal) portfolio which maximises the prospect theory utility function where decision variables are weights of available assets ω subject to constraints on a desirable level of return (in the case of basic prospect theory problem formulation), budget and short sales. This is a nonlinear and non-convex optimisation model as the objective function is nonlinear and non-convex. In order to solve this problem we use heuristics which are an inexact solution approach.

According to the prospect theory portfolio selection problem looks as follows (basic prospect theory model):

$$\text{Maximize } PT_{(x)} = \sum_{s=1}^{S} p_s \;\; v(\sum_{i=1}^{N} r_{si\ w_i}),$$

Subject to the constraints

$$\bar{r}(x) = \sum_{i=1}^{N} \bar{r}_i w_i \geq d,$$

$$\sum_{i=1}^{N} w_i = 1$$

$$w_i \geq 0, \quad i = 1, \dots, N.$$

Studying the prospect theory problem we found that the principle of the model is very similar to that of the index tracking portfolio optimisation problem. The main common feature is that behaviourally based models use a reference point as the limit for desired level of returns in each time period similar to an index tracking model which uses the index as a reference point. Thus it is easy to implement the idea of the index tracking problem into prospect theory by changing the value of the reference point. In this case we let r_0 be a vector of the index value for each time period of the data set not a scalar as it is in the original version of the prospect theory. The following relation can be used for calculating the return on risky asset:

$$R = \mu + \sigma n$$

In this relation *(0, 1), n~N* and the definite return on risk-free asset will be equal to R_f.
The investor's preferences are based on changes in wealth and behavioral biases mentioned in prospect theory. We suppose that the investor has an initial wealth of W0 without any further revenue and profit. In order to manage his/her capital for investing on risky assets and risk-free assets, the investor evaluated the weight of θ for the risky asset and (*1-* θ) fir the risk-free asset $(0 \leq \theta \leq 1)$.

The investors' preferences and decisions depend on the changes in wealth and merge point regarding the utility expected in prospect theory. So, the investor's profit or loss is obtained as the following based on the weight of the risky asset (θ):

$$x = \Delta W$$
$$\therefore x = \left[(1-\theta)W_0\left(1-R_f\right) + \theta W_0(1+R)\right] - W_0$$
$$\therefore x = (1-\theta)R_f + \theta R$$
$$\therefore x = (1-\theta)R_f + \theta(\mu + \sigma n)$$

In the above relation, the investor's initial wealth (W_0) is considered equal to 1.
The probability weighting function proposed for Gerogie et al (2004) is used for this purpose, and it is defined as the following:

$$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1+p)^\gamma)\ ^1/_\gamma}$$

In this relation, γ is the probability weight adjustment coefficient.
In the following table, the values of p and $\pi(p)$ have been compared in terms of $\gamma = 0.8$.
Thus, for the prospect theory regarding behavioral investement strategy we have:

$$V(x) = \begin{cases} \lambda^+ - \lambda^+ e^{-\alpha x} & x \geq 0 \\ \lambda^- e^{\alpha x} - \lambda^- & x \geq 0 \end{cases}$$

Which;
α: indicates the people's general risk aversion coefficient
λ^+: indicates loss aversion coefficient in profit area
λ^-: indicates loss aversion coefficient in loss area

As $\lambda^- > \lambda^+ > 0$, the value function has a higher slope in loss area. *X* that represents for changes in wealth is an indicator of the investors' mental accounting in value function. Then, for the total total areaa surronded by the untility function curve, we have:

$$V(x) = \int_{-\infty}^{+\infty} \upsilon(x) \frac{d}{dx} \pi\big(f(x)\big) dx$$

In the above relation:
$V(x)$: indicates the value function in terms of *x*
$\pi(p)$: indicates the cumulative probability weighting function of *x*
f(x): cumulative distribution function
The following relation is proposed for the considered value:

$$maxV(x) = \int_{-\infty}^{+\infty} \upsilon(x) \frac{d}{dx} \big(f(x)\big) dx$$

So, the following relation is resulted from maximization of the final value function (*V*):

$$V = \int_{-\infty}^{+\infty} \upsilon(x) \frac{d}{dx} \big(f(x)\big) dx$$

Regarding the standard mode of utility function (loss aversion in all possible outputs), the following relation is resulted for the considered value function:

$$V^s = \lambda^+ - \lambda^- e^{\frac{1}{2}\alpha^2 C^2 - \alpha B}$$

Describing the properties of the considered final value function should be done taking partial derivate of the above relations with respect to the risk and return.

3. Numerical Illustrative Results and Conclusion

In the previous chapter we considered the prospect theory and traditional Markowitz approaches. In this section we develop four portfolios from TSE and the portfolio selection is imposed. The outcomes from the selected portfolios are compare. The some main indeces which are observed are risk and returns of each portfolio.

3.1. First Portfolio:

Table of First portfolio: Calculating the optimal weight of portfolio risk assets Standard Model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*
1	0.043837	0.133507	0.191658	0.3907	0.6093	0.02971
2	0.04383	0.109419	0.286394	0.8724	0.1276	0.020883
3	0.129639	0.164166	0.713541	0.9994	0.0006	0.093343
4	0.020278	0.151967	0.051183	0.1123	0.8877	0.0363
5	0.031716	0.014104	0.136244	0.322	0.678	0.03323

Table of First portfolio: Calculating the optimal weight of portfolio risk assets financial behavioral model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*

1	0.01397	0.09798	0.015019	0.14	0.86	0.291133
2	0.01273	0.07255	0.003069	0.2965	0.7035	0.48217
3	0.012877	0.137065	0.02748	0.1569	0.8431	0.493856
4	0.009573	0.066145	-0.2408416	0.19	0.81	0.2408416
5	0.01783	0.11463	0.046502	0.12	0.88	0.226921

3.2. Second Portfolio:

Table of Second portfolio: Calculating the optimal weight of portfolio risk assets Standard Model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*
1	0.06187	0.091988	0.535796	0.999	0.001	0.08972
2	0.03354	0.102512	0.205324	0.6676	0.3324	0.028654
3	0.049244	0.073868	0.497433	0.9995	0.0005	0.084978
4	0.030114	0.168945	0.104259	0.2057	07943	0.03471
5	0.021798	0.079358	0.117167	0.4922	0.5078	0.034155

Table of Second portfolio: Calculating the optimal weight of portfolio risk assets financial behavioral model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*
1	0.01263	0.04272	0.003087	0.50367	0.49633	0.493856
2	0.03683	0.012818	0.189769	0.15	0.85	0.402715
3	0.012797	0.102414	0.002901	0.21004	0.78996	0.49015
4	0.000005	0.0633	-0.19762	0.17	0.83	0.20543
5	0.011414	0.068821	-0.01578	0.186	0.814	0.211016

3.3. Third Portfolio:

Table of Third portfolio: Calculating the optimal weight of portfolio risk assets Standard Model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*
1	0.045406	0.00343	0.290091	0.85245	0.1576	0.021084
2	0.11459	0.152176	0.688149	0.9997	0.0003	0.102124
3	0.01724	0.120062	0.039503	0.13337	0.8663	0.037324
4	0.036651	0.17878	0.135086	0.3122	0.6878	0.026127
5	0.040423	0.14852	0.188011	0.4266	0.5134	0.027253

Table of third portfolio: Calculating the optimal weight of portfolio risk assets financial behavioral model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*
1	0.02151	0.07758	0.006144	0.31659	0.68341	0.492473
2	0.038792	0.11363	0.23147	0.19691	0.80309	0.44063
3	0.011336	0.075796	-0.01536	0.173	0.827	0.346453
4	0.017245	0.127694	0.037156	0.1199	0.8801	0.386612
5	0.02246	0.110246	0.090327	0.8656	0.1344	0.27221

3.4. Forth Portfolio:

Table of forth portfolio: Calculating the optimal weight of portfolio risk assets Standard Model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*
1	0.029864	0.168062	0.103321	0.2157	0.7843	0.033954
2	0.02223	0.08348	0.116551	0.4653	0.5347	0.034187
3	0.046615	0.07116	0.479411	0.999	0.001	0.081638
4	0.031023	0.102752	0.180265	0.5848	0.4152	0.030524
5	0.05645	0.091648	0.479548	0.9991	0.0009	0.07564

Table of forth portfolio: Calculating the optimal weight of portfolio risk assets financial behavioral model

course	μ	σ	Sharpe ratio	$\theta*$	1-$\theta*$	V*
1	-0.00089	0.060407	-0.22167	0.17742	0.82258	0.25207
2	0.011846	0.069656	-0.00939	0.19977	0.80023	0.346571
3	0.027804	0.05011	0.305403	0.5836	0.4164	0.470106
4	0.034036	0.12322	0.17477	0.1345	0.8655	0.28664
5	0.026625	0.077136	0.178262	0.7639	0.2361	0.237169

The results obviosely show the minimized risk with behavioral approach in the certain level of return. But, the return are as the same level and are not significantly different with considration of certain level of risk in the portfolios. Thus, by emlopying the behavioral investment strategy such as Kahneman-Tversky approach, the risk of the portfolio will be minimized neither the return would be the same as the traditional approaches such as Markuwitz is achiving and reporting.

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