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A new behavioral model of rational choice in social dilemma game

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Abstract

In this article, we show that how human decision makers behave in interactive decisions. We interpret the players' behavior with help of the concept of hyper-rationality. These interpretations help to enlarge our understanding of the psychological aspects of strategy choices in games. With help of this concept can be analyzed social sciences and society based on cognitive psychology approach such that human society can be understood easily and predicted more fluently. In addition, we introduce a new game in which there is a dilemma that this dilemma occurs in most societies. We investigate this dilemma based on the claim that each player is hyper-rational. In this dilemma, a weak trust has been created between players, but it is fragile. In many cases, our study provides a framework to move towards cooperation between human decision makers.

Keywords: Game theory, Decision making, Rationality, social dilemma game.

1. Introduction and preliminaries

The theory of rational choice is one one of the fundamental principles which led to generalization and expansion game theory models. The assumption of rational behavior is based on the fact that each player intends to maximum self-interested [1, 4, 6, 9]. Recently Eshaghi and Askari introduced a new concept of rational choice called *hyper-rational choice* [2]. By applying this concept, we propose a model that interprets the interactive conditions among hyper-rational actors. The hyper-rational actor thinks about profit or loss of other actors in addition to his personal profit or loss and then will choose an action which is desirable to him.

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Now consider a rational individual. The set of possible choices of rational individual $i \in \{1, 2, ..., n\}$ is shown with $A_i = \{a_1, a_2, ..., a_n\}$. Given hyper-preferences, how will a hyper-rational individual behave? We assume that given a set of choices $B \subseteq \mathcal{A} = A_1 \times A_2 \times ... \times A_n$. We define the weak hyper-preferences of player i' over the set B as follows:

$$(a_1, a_2, ..., a_n)_i \succeq' (b_1, b_2, ..., b_n)_i \Leftrightarrow either\ a_1 \succeq b_1\ or\ a_1 \preceq b_1\ based\ on\ player\ i' \\ preferences\ for\ player\ 1\ and\ either\ a_2 \succeq b_2\ or\ a_2 \preceq b_2\ based\ on\ player\ i' \\ preferences\ for\ player\ 2\ and\ either\ a_i \succeq b_i\ or\ a_i \preceq b_i\ based\ on\ player\ i' \\ preferences\ and\ either\ a_n \succeq b_n\ or\ a_n \preceq b_n\ based\ on\ player\ i'\ preferences \\ for\ player\ n,$$

where relation $\succeq$ is complete and transitive. We say that $(a_1, a_2, ..., a_n)$ is strictly preferred to $(b_1, b_2, ..., b_n)$, or $(a_1, a_2, ..., a_n) \succ' (b_1, b_2, ..., b_n)$, if $(a_1, a_2, ..., a_n) \succeq' (b_1, b_2, ..., b_n)$ but not $(b_1, b_2, ..., b_n) \succeq' (a_1, a_2, ..., a_n)$. We say the player is indifferent between $(a_1, a_2, ..., a_n)$ and $(b_1, b_2, ..., b_n)$, or $(a_1, a_2, ..., a_n) \sim' (b_1, b_2, ..., b_n)$, if $(a_1, a_2, ..., a_n) \succeq' (b_1, b_2, ..., b_n)$ and $(b_1, b_2, ..., b_n) \succeq' (a_1, a_2, ..., a_n)$. So, we defined set of hyper-preference over set of preferences.

Definition 1.1. *The relation $\succeq'$ on B is* complete *if for all $(a_1, a_2, ..., a_n), (b_1, b_2, ..., b_n) \in B$ either $(a_1, a_2, ..., a_n) \succeq' (b_1, b_2, ..., b_n)$ or $(b_1, b_2, ..., b_n) \succeq' (a_1, a_2, ..., a_n)$, or both.*

Routine behaviors help the person prioritize his/her hyper-preferences and have a belief system about other persons with them. The relation $\succeq'$ is complete. The completeness condition ensures that all outcomes can be compared with each other. In the following, we defined hyper-rationality as follows:

Definition 1.2. *(Hyper-rational) An individual will be called hyper-rational under certainty if is a rational (see Definition 1 in [2]) and their hyper-preferences for preferences (individual or for others) satisfy at least one of the following conditions:*

1. *The player chooses from the set of available alternatives (actions) based on individual preferences;*
2. *The player chooses from the set of available alternatives (actions) based on preferences for other players.*

It can be concluded each hyper-rational player is a rational player, but each rational player is not a hyper-rational player. The hyper-rational player implements the three classes mentioned above to choose an action which is desirable to him. Many human behaviors can be expressed with help of hyper-rationality. Hyper-rational player may choose an action which is not based on his/her benefits but based on the principles which he/she accepts. In other words, based on the hyper-rationality, the player may prefer personal benefits of strategy A to B and B to C but prefer C to A based on the principles which he/she accepts or may not make difference between A and C. Hyper-rationality leads to consideration of the nonmaterial benefits.

In order to describe a game, the set of possible choices of rational individual $i \in \{1, 2, ..., n\}$ is shown with $A_i = \{a_1, a_2, ..., a_n\}$. So, each individual player i has a set of actions A_i available to him and a particular element in the set of actions is denoted by $a_i \in A_i$ [5, 10, 11, 12, 13, 14]. A profile of actions for the players is given by

$$(a) = (a_1, a_2, ..., a_n) \in \prod_{i=1}^{n} A_i$$

or alternatively by separating the action of player i from all other players, denoted by $-i$:

$$(a) = (a_i, a_{-i}) \in (A_i, A_{-i})$$

Finally there are payoff functions for each player i:

$$U_i : A_1 \times A_2 \times ... \times A_n \to \mathbb{R}$$
$$U_i(a) = U_i(a_1, a_2, ..., a_n) = b$$

Now, we apply the hyper-rational choice theory as a basis and main element of modeling in game theory. With help of hyper-rationality, we analyze conditions of a strategic game. In order to describe a game based on concept of hyper-rational choice, the payoff functions for each player i is given by:

$$U_i^j : A_1 \times A_2 \times ... \times A_n \to \mathbb{R}$$

$$U_i^j(a_1, a_2, ..., a_n) = \begin{cases} U_i(a_1, a_2, ..., a_n) & if\ i = j \\ U_j(a_1, a_2, ..., a_n) & if\ i \neq j, \end{cases} \tag{1.1}$$

where U_i^j shows that if player i considers profit (loss) of player j, he will choose an action from a set of available actions which will benefit (lose) player j, for every $i, j \in \{1, 2, ..., n\}$. In other words, based on player i' preferences for player j, he thinks about profit or loss of another player in addition to his personal profit or loss and then will choose an action from a set of available actions which is desirable to him.

Hyper-rationality in game theory helps the player choose successful strategies of the game in interactive conditions and reproduce them. We assume that each player in the game is hyper-rational. Thus, a hyper-rational player will renormalize her opinion based on the common knowledge that each player is hyper-rational. Below, we show the best response functions based on hyper-preferences of players with B, P, and L. Precisely, B_i the best response function of player i based on individual benefit

$$B_i(a_{-i}) := \Big\{ a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(a_i^{'}, a_{-i})\ for\ all\ a_i^{'} \in A_i \Big\}, \tag{1.2}$$

any action in $B_i(a_{-i})$ is at least as good based on individual benefit for player i as every other action of player i when the other players' actions are given by a_{-i}. Precisely, we define the set-valued function K_i by

$$P_i(a_{-i}) := \Big\{ a_i \in A_i : u_{-i}(a_i, a_{-i}) \geq u_{-i}(a_i^{'}, a_{-i})\ for\ all\ a_i^{'} \in A_i \Big\}, \tag{1.3}$$

any action in $K_i(a_{-i})$ for player i relative to every other action of player i is at least the best based on profit for other players when the other players' actions are given by a_{-i}. We call K_i the best response function of player i based on profit for other players. Precisely, we define the set-valued function L_i by

$$L_i(a_{-i}) := \Big\{ a_i \in A_i : u_{-i}(a_i, a_{-i}) \leq u_{-i}(a_i^{'}, a_{-i})\ for\ all\ a_i^{'} \in A_i \Big\}, \tag{1.4}$$

any action in $L_i(a_{-i})$ for player i relative to every other action of player i is at least as good based on the loss for other players when the other players' actions are given by a_{-i}. We call L_i the best response function of player i based on the loss of other players. From relations (1.2), (1.3) and (1.4) we have,

$$B_i(a_{-i}) \cup P_i(a_{-i}) \cup L_i(a_{-i}) = A_i.$$

In competitive interactions, we define strictly dominant action and weakly dominant action based on the loss of other players.

Definition 1.3. *(Strict domination of loss) In a strategic game for player i, action $a_i^{''}$ is strictly dominant on her action $a_i^{'}$ for loss of others, if we have*

$$u_{-i}(a_i^{''}, a_{-i}) < u_{-i}(a_i^{'}, a_{-i}) \quad for\ every \quad a_{-i} \in A_{-i},$$

where u_i is a payoff function that represents player i's preferences. It is defined as strictly dominant action based on benefit for other players similar, but the difference is that direction of the relation $<$ is changed.

Definition 1.4. *(Weak domination of loss) In a strategic game for player i, action $a_i^{''}$ is weakly dominant on her action $a_i^{'}$ for loss of others, if we have :*

$$u_{-i}(a_i^{''}, a_{-i}) \leq u_{-i}(a_i^{'}, a_{-i}) \quad for\ every \quad a_{-i} \in A_{-i}$$

and

$$u_{-i}(a_i^{''}, a_{-i}) < u_{-i}(a_i^{'}, a_{-i}) \quad for\ some \quad a_{-i} \in A_{-i},$$

where u_i is a payoff function that represents player i's preferences. It is defined as weakly dominant action based on benefit for other players similar, but the difference is that direction of relations $\leq$ and $<$ is changed.

The actions which are chosen based on the concept of hyper-rationality (hyper-preferences) and rationality of the players may be similar or different. To prevent ambiguity in interactions, we divide actions of players into three classes as strictly dominant action and weakly dominant action based on individual benefit, strictly dominant action and weakly dominant action based on profit for other players and strictly dominant action and weakly dominant action based on the loss for others. The following proposition shows a method for finding equilibrium in the game.

Proposition 1.5. *The action profile a^* is a equilibrium point of strategic game if and only if hold true in at least one of the following conditions:*

- *Each action of the player is the best response to actions of other players based on personal benefit:*

$$a^* \ \ is \ \ in \ \ B_i(a^*_{-i}) \ \ for \ \ every \ \ player \ \ i,$$

- *Each action of the player is the best response to actions of other players based on the benefit of other players:*

$$a^* \ \ is \ \ in \ \ P_i(a^*_{-i}) \ \ for \ \ every \ \ player \ \ i.$$

- *Each action of the player is the best response to actions of other players based on loss of other players:*

$$a^* \ \ is \ \ in \ \ L_i(a^*_{-i}) \ \ for \ \ every \ \ player \ \ i.$$

We consider equilibrium based on concept of Nash [7, 8]. In the next section, we introduce a new game and analyze it using the concept of rationality and the concept of hyper-rationality.

2. Rostam's Dilemma (Weak Trust Game)

Rostam's Dilemma is one of the other several dilemmas that we confront in attempt to achieve cooperation. In this game, cooperation can produce the best possible overall result, but there is a non-cooperative Nash equilibrium that wants to draw us toward itself. Difference between this dilemma and some of the other dilemmas is that in the Rostam's Dilemma, cooperation is weak dominant over non-cooperation. Despite Prisoner's Dilemma that players have no trust in each other, in Rostam's Dilemma, there is a weak trust between players but yet this weak trust doesn't ensure that cooperation completely. There has been a view that if players trust in each other, they will obtain better result but this game shows that despite players' weak trust in each other, there is no definite guarantee to achieve a desirable result and this trust is fragile.

Abolghasem Ferdowsi Toosi is an Iranian epic poet and composer of Iran national epic Shahnameh that stated conflict between Rostam and Sohrab in the epic form [3]. Rostam and Sohrab epic is one of the saddest events of Shahnameh. Rostam is one of the Iranian athletes that marry Samangan king's daughter, Tahmineh. Some days after marriage, he said goodbye to Tahmineh and happily came to Iran from there went to Zabolestan. After nine months Tahmineh gave birth to a boy and informed Rostam. He was named Sohrab. One day Sohrab went to his mother and said: who is my father? If someone asks me what I say in answer? Mother said: you are the son of robust athlete Rostam and from Sam and Zal race.

In fact, there are several intertwined pieces in Rostam and Sohrab story but the main conflict is between Rostam and Sohrab. After some years Soharb with an army of Tooranian and Samanganian depart for war with Iranians. When Rostam reached to Sohrab on the battle plain said: let go from here to another side and fight. Sohrab agreed and demanded person to person war and said: you are old and not able to fight against me. Rostam said: calm down. Many demons were killed in my hand so wait to see me in the fight. I don't like to fight with you and kill you. Sohrab suddenly asked: who are you and from what race? I think you are Rostam. Rostam said: no I'm not. Both went to the battle field and fought to the end of the day. Rostam said it is night, tomorrow we wrestle. Rostam said himself: I have a son from Tahmineh who is a youth as old as Sohrab and maybe he is himself.

By sunrise, they went again to the battle field. Sohrab said to himself: the more I watch him, the more I think that he is Rostam himself and I have not to fight with him. Sohrab said to Rostam: how was last night? Come on sit down to speak with each other and don't fight. My heart is drawn to you. I asked to know your name a lot of time but no one told me your name then don't hide your name. Rostam said: last night, we talked about wrestling you can't deceive me. Then they wrestled and fought for a while finally Sohrab took Rostam's belt and threw him on the ground and took out dagger but Rostam said: our tradition is that one who throws a person on the ground doesn't kill him the first time but in the second time kill him. Sohrab agreed because he was both brave and chivalrous. They went again to the battle field and grappled, this time Rostam threw Sohrab on the ground then took out his dagger and killed him.

With help of game theory, the game between Rostam and Sohrab is modeled. Rostam has two actions; either he says (S) his name to Sohrab or not says (NS) his name to Sohrab. Sohrab also has two actions; either he fights (F) against Rostam or not to fight (NF) against Rostam. We call this game as Rostam's Dilemma. In Rostam's Dilemma, Sohrab is considered as row player (player 1) and Rostam is considered as column player (player 2)(Fig. $1g_2$).

The above game is a symmetric game. Based on concept of rationality player 1 has weak dominant action NF, and player 2 has weak dominant action S. This game has two Nash equilibria (NF, S) and (F, NS). Rostam's Dilemma is strategy maker of order $(2, 2)$. Moreovere, the game has four

g_2	S	NS
NF	4,4	1,3
F	3,1	1,1

g_3	C	B
C	4,4	1,3
B	3,1	1,1

Figure 1: $G^{'}$ is a Rostam's Dilemma. $G^{''}$ is a game between wife and husband.

pairs of rational actions $(NF,S)_{1,2}$, $(F,NS)_{1,2}$, $(NF,NS)_2$ and $(F,S)_1$. In Prisoner's Dilemma based on classical rationality, player 1 prefers $(NF,S)_1 \succeq^{'} (F,S)_1 \succeq^{'} (NF,NS)_1 \sim^{'} (F,NS)_1$.

In the concept of hyper-rationality, the player thinks about profit or loss of other players in addition to his personal profit or loss and then will choose an action which is desirable to him [2].

1. Each player is thinking of making a profit to another player.

In game g_2, for player 1 we have: based on concept of hyper-rationality, given fixed S for player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose NF (player 2, earns a reward 4), it can conclude that pair of action (NF,S) is chosen. With choosing NS by player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose NF (player 2, earns a reward 3), it can conclude that pair of action (NF,NS) is chosen. Therefore, for the player 1 (Sohrab), based on the profit of another player, NF is a strictly dominant action.

In game g_2, for player 2 we have: based on concept of hyper-rationality, given fixed NF for player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose S (player 1, earns a reward 4), it can conclude that pair of action (NF,S) is chosen. With choosing F by player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose S (player 1, earns a reward 3), it can conclude that pair of action (F,S) is chosen. So, for the player 2 (Rostam), based on the profit of another player, S is a strictly dominant action.

2. player 1 is looking to profit for player 2 and player 2 seeks to lose of player 1.

In game g_2, for player 1 we have: based on concept of hyper-rationality, given fixed S for player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose NF (player 2, earns a reward 4), it can conclude that pair of action (NF,S) is chosen. With choosing NS by player 2, we can see that if player 1, seeks to incur profit to his opponent, he will choose NF (player 2, earns a reward 3), it can conclude that pair of action (NF,NS) is chosen. Therefore, for the player 1 (Sohrab), based on the profit of another player, NF is a strictly dominant action.

In game g_2, for player 2 we have: based on concept of hyper-rationality, given fixed NF for player 1, we can see that if player 2, seeks to incur loss to his opponent, he will choose NS (player 1, earns a reward 1), it can conclude that pair of action (NF,NS) is chosen. With choosing F by player 1, we can see that if player 2, seeks to incur loss to his opponent, he will choose NS (player 1, earns a reward 1), it can conclude that pair of action (F,NS) is chosen. So, for players 2 (Rostam), based on the loss of another player, NS is a strictly dominant action.

3. Each player is thinking of making a loss to another player.

In game g_2, for player 1 we have: based on concept of hyper-rationality, given fixed S for player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose F (player 2, earns a reward 1), it can conclude that pair of action (F,S) is chosen. With choosing NS by player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose F (player 2, earns a reward 1), it can conclude that pair of action (F,NS) is chosen. Therefore, for the player 1 (Sohrab), based on the loss of another player, F is a strictly dominant action.

In game g_2, for player 2 we have: based on concept of hyper-rationality, given fixed NF for player

1, we can see that if player 2, seeks to incur loss to his opponent, he will choose NS (player 1, earns a reward 1), it can conclude that pair of action (NF, NS) is chosen. With choosing F by player 1, we can see that if player 2, seeks to incur loss to his opponent, he will choose NS (player 1, earns a reward 1), it can conclude that pair of action (F, NS) is chosen. So, for the player 2 (Rostam), based on the loss of another player, NS is a strictly dominant action.

4. Player 1 is looking to loss to player 2 and player 2 seeks to profit for player 1.

In game g_2, for player 1 we have: based on concept of hyper-rationality, given fixed S for player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose F (player 2, earns a reward 1), it can conclude that pair of action (F, S) is chosen. With choosing NS by player 2, we can see that if player 1, seeks to incur loss to his opponent, he will choose F (player 2, earns a reward 1), it can conclude that pair of action (F, NS) is chosen. Therefore, for players 1 (Sohrab), based on the loss of another player, F is a strictly dominant action.

In game g_2, for player 2 we have: based on concept of hyper-rationality, given fixed NF for player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose S (player 1, earns a reward 4), it can conclude that pair of action (NF, S) is chosen. With choosing F by player 1, we can see that if player 2, seeks to incur profit to his opponent, he will choose S (player 1, earns a reward 3), it can conclude that pair of action (F, S) is chosen. So, for players 2 (Rostam), based on the profit of another player, S is a strictly dominant action.

This concept explains that, based on the loss of player 2, F is a strictly dominant action for player 1 and based on the loss of player 1, NS is a strictly dominant action for player 2. If interaction between players is based on loss of another player, player 1 prefers: $(F, NS)_1 \sim' (F, S)_1 \succeq' (NF, NS)_1 \succeq' (NF, S)_1$, and player 2 prefers: $(F, NS)_2 \sim' (NF, NS)_2 \succeq' (F, S)_2 \succeq' (NF, S)_2$. Therefore, for both players pair of actions (F, NS) is hyper-rational. On other hands, If interaction between players is based on profit of another player, player 1 prefers: $(NF, S)_1 \succeq' (NF, NS)_1 \succeq' (F, NS)_1 \sim' (F, S)_1$, and player 2 prefers: $(NF, S)_2 \succeq' (F, S)_2 \succeq' (NF, NS)_2 \sim' (F, NS)_2$. Therefore, for both players pair of actions (NF, S) is hyper-rational.

Rostam's game put a dilemma on the way of players. Based on concept of rationality player 1 has weak dominant action NF, and player 2 has weak dominant action S. Although players have weak dominant action, they select weak dominated action and obtain the least payoff in game. In the other words, the player's weak dominant action generates a weak trust between players, but this weak trust is fragile. In the Rostam's dilemma, weak trust between players was broken and both selected weak dominated action. As mentioned above, taxonomy of player's hyper-preferences depends on environmental condition, the kind of behavior interactive, self-evaluation system and evaluation system of other interacting persons. Taxonomy of player's hyper-preferences has six main behavioral options: individual profit, individual loss, profit for others and loss for others and indifferent between profit and loss for others.In other words, based on collective loss thinking, both player prefers: $(F, NS) \succeq' (NF, S) \succeq' (F, S) \succeq' (NF, NS)$ or $(F, NS) \succeq' (NF, S) \succeq' (NF, NS) \succeq' (F, S)$. Therefore, Weak trust between Rostam and Sohrab was broken and turned into distrust to each other and son (Sohrab) was killed by father(Rostam).

2.1. Application of Rostam's Dilemma

Now, as an application of the Rostam's Dilemma, the social dilemma in the way of a wife and husband is modeled. Consider a couple who live together for several years, The couple is disagreeing with each other about a problem. This dispute leads to having trouble between husband and wife in some other problems. If both player condonation C of these disputes and resolve all problems, then each earns reward 4. If the husband select condonation of disputes, but the wife blame husband and intensify disputes B, then the wife gain payoff 3 and the husband gets 1 and vice versa. If the wife to blame her husband and the husband to blame her wife, more intensify disputes becomes, then both gain a payoff 1. The wife is considered as row player (player 1) and husband is considered as column (player 2)(Fig. $1g_3$).

This dilemma occurs in most societies, wife, and husband have weakly dominant strategy C, but sometimes wife and husband select weakly dominated strategy B, which lead to intensifying disputes. The game Nash equilibria are (C, C) and (B, B). Eshaghi and Askari introduced a new concept that Called *taxonomy* [2]. Taxonomy of hyper-preference means that if we face a player with two choices of hyper-preferences, she will necessarily have an opinion on which she likes more. Taxonomy of player's hyper-preferences depends on environmental condition, the kind of behavior interactive, self-evaluation system and evaluation system of other interacting persons, helps to the wife and husband that based on collective profit prefer (C, C) and based on collective loss prefer (B, B).

3. conclusion

In this paper, using the concept of hyper-rationality we examined an important social dilemma. The concept of hyper-rationality allowing a player to thinks about profit or loss of other players in addition to his personal profit or loss and choosing an action which is desirable to him. The concept of hyper-rationality extends the advantages of game theory and can be applied instead of the theory of rational choice in social sciences and society can be analyzed based on cognitive psychology approaches so that human society can be understood easily and predicted more fluently.

Moreover, we introduce a new game in which there is a dilemma that this dilemma occurs in most societies. We investigate this dilemma based on the claim that each player is hyper-rational. Difference between this dilemma and some of the other dilemmas is that in the Rostam's Dilemma, cooperation is weak dominant over non-cooperation. There has been a view that if players trust in each other, they will obtain better result but this game shows that despite players' weak trust in each other, there is no definite guarantee to achieve a desirable result and this trust is fragile

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